Supersonic Turbulent Boundary-Layer Separation Control Using a Morphing Surface

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Separated flows arising due to shock-wave/turbulent boundary-layer interactions can cause problematic low-frequency unsteadiness with potentially severe structural response. High-fidelity large-eddy simulations are employed to examine surface morphing as a way to reduce the size of the separation region, and thus favorably alter the unsteadiness characteristics. The configuration considers a turbulent Mach 2.7 flow at a Reynolds number of $Re_{\infty} = 54,600$ subjected to an impinging shock system with a pressure ratio of $p_2/p_1 = 3$, which results in separation and the presence of structurally relevant low-frequency unsteadiness. The control surface, centered about the shock impingement location and extending over the separation region, is allowed to deform under material property-based realizability constraints until an asymptotic state is achieved. The criterion for deformation uses a measure proportional to the directional surface shear stress. At an asymptotic state, the deformed surface reveals a shape consistent with aerostructural optimization and a maximum height of $0.32\delta_{\infty}$. Control mitigates the sharp initial pressure gradient of the uncontrolled flow to delay and reduce separation extent (by 50%) with diminution of low-frequency content and turbulent kinetic energy. Modal decomposition highlights these effects in the energy content of the prominent modes. Morphing may thus provide a means to adjust the local surface deflection in a manner that reduces some of the problems associated with turbulent separation.

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I. Introduction

SHOCK-WAVE boundary-layer interactions (SWBLIs) can significantly affect the performance of high-speed vehicles [1]. The imposed adverse pressure gradient can greatly alter the overall flowfield structure by inducing features such as separation, vortical structures, additional shocks, and expansions that are absent in the inviscid counterpart and the introduction of new coherent unsteadiness scales. From an engineering standpoint, SWBLIs can yield localized peaks in thermomechanical variables (such as skin friction, heat transfer, and pressure), enhance total pressure losses and distortion, reduce control authority, and potentially trigger large-scale events such as engine unstart. Low-frequency flow features may also reduce structural life, generate undesirable noise, and promote failure of the attached components [2].

The properties of SWBLIs depend on flow as well as geometric parameters, including Mach and Reynolds numbers, the nature of the shock generator and the boundary layer, which can be laminar, transitional or turbulent. Canonical two-dimensional interactions (such as those produced by compression ramps or shocks impinging on a boundary layer) and three-dimensional (3-D) interactions (such...
as those produced by fins at an angle of attack) have proven very useful in examining the salient features of SWBLIs [3]. The present work is focused on understanding morphing-based control of an oblique shock wave interacting with a turbulent boundary layer; the primary parameters, chosen based on an experimental database displaying flow separation, are delineated in the following. The main features of the interaction in such impinging shock turbulent SWBLIs are the separation bubble between the separation and reattachment points and a reflected shock system formed by coalescence of the compression and expansion waves [4].

The characteristic low-frequency unsteadiness or breathing of SWBLIs has been studied extensively because of its adverse structural response implications. Some relatively recent efforts using advanced measurements and simulation techniques may be found in the works of Clemens and Narayananswamy [5], Agostini et al. [6], Adler and Gaitonde [7], and Arora et al. [8]. The underlying causes of the observed low-frequency content have been attributed to different mechanisms whose details may depend on the Reynolds number under consideration. One mechanism attributes the unsteadiness to the influence of disturbances in the incoming boundary layer [9–11].

The second assigns an intrinsic mechanism associated with the separation bubble progression relative to the oblique shock. These are often associated with underlying amplifier and oscillator stability types, respectively. Recent turbulent SWBLI efforts have shown the influence of streamwise-oriented counter-rotating Görtler-like vortices [16,17] that are more easily discerned in transitional SWBLI conditions [18,19]. These vortices may interact strongly with other unsteadiness mechanisms in SWBLIs [12], such as the oscillator mechanism of unsteadiness [16].

Several of the different strategies of SWBLI control have been summarized by Déllery and Bur [20] and Gaitonde [1]. Perhaps the most common active control technique considers suction (bleed) through wall cavities to either remove the low-energy fluid in the boundary layer, which is particularly susceptible to separation under adverse pressure gradient conditions, or enhance its energy through suitably oriented blowing [21,22]. A passive control combining the merits of both the suction and blowing strategies is a cavity closed with a perforated plate located at the shock impingement location [23]. In recent years, vortex generators placed upstream of the SWBLI have seen increased interest [24]; these seek to transfer momentum from the outer layer of the boundary layer toward the wall. Likewise, a small deformation of the wall near the shock impingement location, also known as a shock control bump (SCB), essentially introduces a more gradual compression upstream of the shock impingement. The form is similar to a separation bubble but substantially minimizes the wave drag with a relatively small effect on the viscous drag (see the work of Bruce and Colliss [25]). Other passive controls include wall temperature modifications, which affect the shape factor of the incoming boundary layer [23].

The performance of passive control techniques deteriorates under offdesign conditions. For instance, significant changes in the shock impingement location from the design-optimal case may incur additional penalties, and SCBs may result in higher wave and friction drag [20]. Active controls can lift this constraint but require energy and additional onboard components to maintain their operation. This puts forth the potential of a technique that forms the focus of the present effort: specifically, altering SWBLI characteristics with adaptive surface deformation or surface morphing. The advantages of compliant materials over other approaches are their effectiveness and simplicity of concept [25,26].

Early experimental and theoretical works [27–29] achieved drag reduction by reducing growth rates of Tollmien–Schlichting waves to delay transition. Davies and Carpenter [30] and Carpenter [31] showed theoretically that transition can be delayed indefinitely for a particular compliant surface configuration and a low level of freestream turbulence. Compliant surfaces have yielded nominal reductions in drag for transitional and turbulent flows [32–34]. Such techniques are increasingly being explored to control aerospace related flows; an interesting recent example is the work of Jinks et al. [35], who performed numerical simulations of an adaptive SCB for transonic airfoils to demonstrate the advantages over conventional fixed bumps and to highlight various practical considerations related to active surface deformation. The underlying materials technology shows considerable promise, and it is the subject of vigorous recent and current efforts (see, for example, the works of Duerig et al. [36], Karaca et al. [37], and Chen et al. [38]).

Of the many SWBLI-related issues noted previously, the present work seeks to reduce the low-frequency content that triggers adverse structural responses. Many prior studies have examined fluid–structural interactions [39–41], aeroelastic stability [42,43], coupling [44], and acoustic radiation [45,46]. Although the results indicate the potential of suitably placed compliant surfaces to control SWBLIs, relatively little effort has been placed on examining the underlying mechanisms and the implementation of morphing surfaces for SWBLI control. A notable exception is the recent experimental investigation [47]. A compliant surface with a soft material (urethane rubber) displayed significant differences from rigid materials in terms of separation location and energy content in shock oscillations. However, the time-averaged lift and drag forces showed relatively little change between the compliant and rigid surfaces.

In our previous efforts [48,49], both passive SCBs and active surface morphing were explored for the situation where a laminar boundary layer encounters impinging shock-initiated transition. The morphing surface effectively inhibited transition and eliminated separation-related unsteadiness by modulating the relatively sharp uncontrolled SWBLI pressure rise at separation and shock-impingement locations. An added observation was a lower specific entropy rise with control, with accompanying effects on total pressure. The present work examines the more practically important fully turbulent SWBLI, i.e., with an incoming equilibrium turbulent boundary layer. The state of the incoming boundary layer has a substantial effect on flow features; in particular, the uncontrolled shock-induced transitional flow of Shinde et al. [49] displays prominent Görtler-like vortices downstream of the impingement point with a distinct spanwise periodicity. As such, the separation and reattachment regions are relatively distinct, and they facilitate a more straightforward implementation and convergence of the control technique. The separation region in turbulent SWBLIs, on the other hand, has relatively more robust characteristics (see the works of Gaitonde [1] and Dolling [3]).

The present work thus considers active surface control methodology for this more complex fluid dynamics: again, with the immediate goal of reducing low-frequency unsteadiness through separation control. To this end, we use large-eddy simulations (LESs) corresponding to the experimental flow configuration of Bo et al. [50]. The free-stream Mach number is 2.7, and the Reynolds number is based on an inflow boundary-layer thickness of 54,600. The numerical method is described in Sec. II.A. The control surface morphing technique follows the approach of Shinde et al. [49], as described in Sec. II.B. In addition to the criterion for surface deformation, Sec. II.B also explains the procedure by which the aerostructural framework uses the local reversed flow criterion to inform the deformation, which is constrained to elastic ranges with a structural integrity analysis. For concreteness, the structural failure analysis is performed using the material properties of maraging steel, American Iron and Steel Institute (AISI) grade 18Sn (300). This choice is motivated by our previous effort [49], where its properties in terms of the von Mises stress with reference to the yield stress were favorably compared with aluminum alloy Al-7075-T6. Section III describes the experimental and LES flow configuration as well as the control surface specifications. The results are presented in Sec. IV for both uncontrolled and controlled SWBLIs. The properties of the incoming boundary layer are important to the analysis, and they are discussed in Sec. IV.A. The effects of morphing are then discussed in terms of coherent structures and separation (Sec. IV.B) as well as unsteadiness (Sec. IV.C), whereas comments on the structural integrity are provided in Sec. IV.D, followed by brief conclusions.

II. Theoretical and Numerical Model

A. Governing Equations

The equations describing the flow are the compressible Navier–Stokes equations:
written in strong conservation form using curvilinear coordinates \( \xi, \eta, \zeta \), and \( r \) with Jacobian \( J = \partial (\xi, \eta, \zeta, r) / \partial (x, y, z, t) \). The vector of conserved variables is \( \mathbf{U} = (\rho, \rho u, \rho v, \rho w, \rho E) \), with \( \rho, u, v, w, \) and \( E \) denoting the density, the Cartesian components of the velocity, and the internal energy \( E \), respectively. \( F, G, \) and \( H \) are the inviscid fluxes; and \( F_v, G_v, \) and \( H_v \) are the viscous fluxes. For example, \( F \) for \( u \) is given by

\[
\frac{\partial}{\partial t} \left( \frac{U}{J} \right) + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} + \frac{\partial H}{\partial z} = \frac{1}{Re} \left[ \frac{\partial F_v}{\partial x} + \frac{\partial G_v}{\partial y} + \frac{\partial H_v}{\partial z} \right]
\]

(1)

with corresponding expressions for the other flux vectors (see the work of Shinde et al. [19]). Here, the contravariant velocity \( \mathbf{U} = \xi_u + \xi_v u + \xi_w \) and \( \xi_v \) and \( \xi_w \) are the viscous fluxes. For example, \( F \) for \( u \) is given by

\[
F = \frac{1}{J} \left( \begin{array}{c}
\rho U \\
\rho u U + \xi_x p \\
\rho v U + \xi_y p \\
\rho w U + \xi_z p \\
\left( \rho E + p \right) U - \xi_p p
\end{array} \right)
\]

(2)

\( F_v \) for \( u \) is given by

\[
F_v = \frac{1}{J} \left( \begin{array}{c}
\xi_x U \\
\xi_x u U + \xi_y \tau_{x y} + \xi_z \tau_{x z} \\
\xi_x v U + \xi_y \tau_{x y} + \xi_z \tau_{y z} \\
\xi_x w U + \xi_y \tau_{x y} + \xi_z \tau_{y z} \\
\xi_x b_x + \xi_y b_y + \xi_z b_z
\end{array} \right)
\]

(3)

The shear-stress tensor and conduction heat transfer terms are given by

\[
\tau_{x y} = M_{\infty} \left[ \frac{\partial u_l}{\partial x_l} + \frac{\partial u_k}{\partial x_k} + \lambda_b \frac{\partial u_k}{\partial x_l} \delta_{lk} \right]
\]

\[
\dot{q}_l = - \frac{M_{\infty} \lambda_b}{Pr(\gamma - 1)} \frac{\partial^2}{\partial x_l^2} \delta_{lk}
\]

(4)

The dynamic viscosity of the fluid, modeled with Sutherland’s law, is \( \mu \); and the Stokes hypothesis for the bulk viscosity (\( \xi_b = -2/3\mu \)) is assumed. The flow conditions allow for the use of the perfect gas assumption of \( p = \rho T / M_{\infty}^2 \) and a constant value of the Prandtl number of \( Pr = 0.72 \). Flow variables are nondimensionalized by their reference (\( \infty \)) values, except for pressure, which is normalized by \( \rho_{\infty} U_{\infty}^2 \). The length scale is chosen to be the boundary-layer thickness at the domain inflow \( \delta_{\infty} \). Thus, \( Re = \rho_{\infty} U_{\infty} \delta_{\infty} / \mu_{\infty} \).

The system is discretized using a mesh as described in the following. A high-order Pade-type compact sixth-order finite difference filter is employed together with an eighth-order implicit low-pass filter (\( \alpha_l = 0.49 \)), which provides the required numerical stability and serves as an implicit subgrid model [51, 52]. Shock regions are detected using a specified threshold with a simple switch [53]; in these, the high-order compact scheme is replaced by the classical Roe scheme with a second-order reconstruction. For numerical efficiency, an implicit time-marching method is used (Beam and Warming [54]) with two Newton-like subiterations to reduce factorization and explicit boundary condition application errors. Further details on the time scheme, together with validation studies, may be found in the works of Gaitonde and Visbal [55] and Visbal and Gaitonde [56].

### B. Morphing Approach and Structural Considerations

The morphing approach followed is generally similar to that discussed in Ref. [49]. The control surface extent on either side of the shock impingement location is specified in Sec. II.A.

The trigger for deformation is assumed to derive from a desirable feature that is to be controlled: in this case, separation. The near-wall flow velocity is a direct measure of flow separation, and it is readily accessible in the numerical simulations; in practice, a skin-friction measurement can provide an estimate of the near-wall flowfield. Thus, the procedure monitors the near-wall streamwise flow velocity, which is proportional to the streamwise component of the shear force. When negative values are encountered, the control surface deforms dynamically into the flow in the wall-normal direction, with reverse motion for positive values. The degree of deformation is taken to be proportional to the magnitude of the flow velocity; the dynamic deformation of the control surface \( y_c \) is thus specified to be

\[
y_c = \begin{cases} 
|u_r|/r dt & \text{for } u_r < 0 \\
-|u_r|/r dt & \text{for } u_r > 0 
\end{cases}
\]

(5)

where \( u_r \) and \( u_r \) are the velocity vector and streamwise velocity near the control surface. Velocities measured at a distance of 0.01\( \delta_{\infty} \) above the control surface have been found suitable for the control scheme. In this manner, a true or surrogate wall stress measurement, which provides a velocity gradient, can be employed to extract a near-wall velocity scale for feedback.

The time step \( dt \) is synchronized with the time-step size of the flow simulation. Turbulent fluctuations in the near-wall region can incur large, structurally unrealizable deformations and result in highly distorted meshes that amplify numerical discretization error. This is precluded by incorporating a smooth higher-order polynomial fit of the 12th order in the streamwise direction to modulate the control surface deformation by following a Gauss–Newton nonlinear least-squares procedure. Additionally, an exponential damping function ensures a clamped boundary at the edges of the deformation region. The method of Gauss–Newton nonlinear least squares is elaborated on in the Appendix. Since the flow is nominally two-dimensional, the spanwise direction is relatively benign; in this direction, the control surface deformation is kept uniform.

The parameter \( r \) in Eq. (5) is inserted into the formulation to control the sensitivity of the structural deformation to the flow velocity \( u_r \). When \( r = 0 \) set to zero, the control becomes inactive or passive. An operational, practically appropriate value of \( r \) can be specified only after the local flow scales are established. The transitional interaction of Shinde et al. [49] represented a relatively less robust interaction, and \( r = 1.0 \) was determined to be a suitable value. In the present fully turbulent simulations, a higher \( r \) becomes necessary. In the present case, \( r = 10.0 \) is employed to ensure a gradual deformation of the control surface; this value is updated based on a measure of the separation (\%s) as \( r = r^0 \times 10^s \). The technique then decelerates the control surface response toward an equilibrium position.

To ensure that the morphing surface does not exceed elastic limits of the material, a concurrent structural analysis is performed using a finite element approach. Specifically, the commercial solver Abaqus [57] is integrated into the solution procedure to provide an implicit dynamic analysis using direct integration. An extension of the second-order Newmark-\( \beta \) method [58, 59] is used. The required model parameters of \( \alpha_1, \beta_\) are set to \( -0.05, 0.275625, \) and 0.55, respectively. At each flow iteration, the stresses are computed by transferring the time-evolving control surface deformation and velocity to Abaqus. The prescribed motion can be accompanied or replaced by the actuation force and aerodynamic pressure. Forces are not necessary in the current setting, and Abaqus is used to primarily ensure that the deformation does not exceed the ultimate yield stress.

The flowchart of the aerostructural solver, taken from the work of Shinde et al. [49], is shown in Fig. 1. Briefly, the governing equations of Sec. II.A together with initial and boundary conditions of Sec. III are solved by the aerodynamic solver. Flow separation regions are identified using negative streamwise velocity regions in the near-wall region. The control surface deformation is then predicted using Eq. (5). The deformation is provided as input to the structural solver (Abaqus), which returns the von Mises equivalent stress (discussed in the following) as a fraction of the ultimate yield stress for the predicted shape of the control surface. When the fraction is higher...
than the chosen threshold, the morphing procedure (the flow solver) continues with the current shape to avoid plastic deformation. Although the design of the control technique and the selection of the material properties are performed prior to the start of the simulation, the optimization loop of Fig. 1 is performed concurrently with the simulation.

The von Mises stress criterion is indicative of the maximum distortion energy, and it provides the deformation-induced structural stress. The value can be compared with the yield strength \( \sigma_y \) of the material to ensure operation in the elastic regime. The von Mises stress is estimated as

\[
\sigma_v = \sqrt{\frac{3}{2} s_{ij} s_{ij}}
\]

where \( s_{ij} \) is the deviatoric stress, which is expressed as \( s_{ij} = \sigma_{ij} - (1/3)\sigma_{ii}\delta_{ij} \), where \( \sigma_{ij} \) is the Cauchy stress tensor and \( \delta_{ij} \) is the Kronecker delta.

### III. Flow Configuration and Simulation Details

The simulations are patterned after experiments performed by Bo et al. [50] in a low-noise Mach 2.7 wind tunnel at a unit Reynolds number of \( Re_l = 8.79 \times 10^8 \), as well as the operational stagnation pressure and temperature of \( P_0 = 1.0 \times 10^5 \) N/m\(^2\) and \( T_0 = 300 \) K, respectively. Figure 2 shows a schematic of the tunnel together with the computational domain. The \( 200 \times 200 \times 400 \) mm\(^3\) test section includes large optics windows of size \( 200 \times 400 \) mm on all four walls for optical measurements. The experiments were conducted on a toughened glass wall with a sharp leading edge, which extends into the nozzle for \( \approx 110 \) mm. A fully turbulent boundary layer was obtained by implementing transition bands on the extended part of the glass wall. A steady incident shock was generated by using a full shock generator with a flow deflection angle of \( \alpha = 10.5 \) deg. The schematic of Fig. 2 displays the steel shock generator, which spans over the entire test section. Furthermore, it consists of a toughened glass window of size \( 200 \times 100 \) mm for the optical access.

The inflow boundary-layer thickness, measured using particle image velocimetry (PIV), is \( \delta_{in} \approx 6.3 \) mm, which corresponds approximately to the upper extreme \( \delta_{100} \) of the turbulent boundary layer entering the viewing region; whereas the momentum thickness and corresponding Reynolds number are \( \theta = 0.665 \) mm and \( Re_\theta = 5854 \), respectively. The main flow parameters are summarized in Table 1.

The schematic of the configuration depicted in Fig. 2 also includes the LES computational domain (marked by red boundaries), and displays a numerical schlieren for reference. The LES domain lengths in the streamwise and spanwise directions are \( L_x/\delta_{in} = 40 \) and \( L_z/\delta_{in} = 6 \), respectively; whereas in the wall-normal direction, \( L_y/\delta_{in} = 12 \) at the inflow and \( L_y/\delta_{in} = 9.68326 \) at the outflow encompass the features of interest. The shock generator (with an inclination of \( \alpha = 10.5 \) deg) extends in the streamwise direction from the origin \( (x = 0) \) of the LES domain to \( x = 12.5 \delta_{in} \). The top boundary, including the shock generator, is modeled as an inviscid wall. The bottom boundary is an adiabatic wall with a no-slip velocity condition. For the control case, the bottom wall includes a control surface situated at the shock foot (refer to schematic in Fig. 3) in the same manner as in the work of Shinde et al. [48]. The outflow uses a Neumann boundary condition with a linear extrapolation of all variables, whereas the periodicity boundary condition is enforced in the spanwise \( z \) direction.

The computational domain is discretized in the streamwise, wall-normal, and spanwise directions \( (n_x \times n_y \times n_z) \) using \( 1201 \times 279 \times 241 \) grid points, respectively, with the total grid size of \( \approx 81 \times 10^6 \). The streamwise and spanwise directions are uniformly discretized; whereas the grid in the wall-normal direction is stretched away from the bottom wall with geometric progression using a growth ratio of 1.03, which leads to 140 points inside the inflow boundary-layer thickness \( \delta_{in} \). The grid resolutions in the wall units in the streamwise and spanwise directions are \( \Delta x^+ \approx 12.82 \) and \( \Delta z^+ \approx 9.62 \), respectively. In the wall-normal direction, the grid resolution is \( \Delta y_{min}^+ \approx 489 \) near the top boundary and \( \Delta y_{max}^+ \approx 23 \) near the bottom boundary.

#### Table 1 Principal flow parameters

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<th>Parameter</th>
<th>Value</th>
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<td>( M_\infty )</td>
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<tr>
<td>( P_0 )</td>
<td>( 1.0 \times 10^5 ) N/m(^2)</td>
</tr>
<tr>
<td>( T_0 )</td>
<td>300 K</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>10.5 deg</td>
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<tr>
<td>( \delta_{in} )</td>
<td>6.3 mm</td>
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<tr>
<td>( Re_\theta )</td>
<td>54,614</td>
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![Fig. 1 Flowchart of aerostructural coupling procedure. From the work of Shinde et al. [49].](image1)

![Fig. 2 Schematic of the experimental setup with superposed LES computational domain.](image2)

![Fig. 3 Schematic of flow configuration with control surface.](image3)
0.19 at the wall, $\Delta y^+ \approx 12.82$ at the inflow boundary-layer height, and $\Delta y_{\text{max}}^+ \approx 40$ at the top extremum.

The inflow boundary-layer profile is an important parameter in the simulation. In the present simulations, the digital filter procedure originally proposed by Klein et al. [60], and later improved by Xu et al. [61] and Touber and Sandham [62], is employed. The method limits the required filtering operation to a two-dimensional inflow plane while using a temporal correlation function to avoid three-dimensional filtering. Furthermore, the spatial autocorrelations are assumed to be exponential rather than Gaussian, which leads to better correlations for the dominant flow structures. The streamwise mean velocity and Reynolds stresses for the digital filtering were obtained from the Mach 3 Reynolds number $Re_{\delta_m} = 49211$, Direct Numerical Simulation (DNS) database of Bernardini and Pirozzoli [63]. In the LES, the length scales are normalized by the reference length scale $\delta_m$, and thus the DNS statistics were directly used without any scaling of the profiles. The procedure requires about $8 \sim 12\delta_m$ distance downstream to establish [64]. In the present work, a suitable equilibrium profile is obtained at a distance of $8.5\delta_m$ from the inflow. The LES are performed for a total time duration of $t_{\text{end}}/\delta_m = 1000$ with a time-step of $\Delta t/\delta_m = 0.001$. The statistics are gathered at a sampling rate of $100\Delta t$ after a statistically stationary state is reached; this requires an initial transient time of $tU_0/\delta_m = 100$, corresponding to a flowthrough time of $\approx 2.5$.

The schematic of Fig. 3 shows the placement of the control surface, anticipating the uncontrolled flow separation region to be approximately centered around the shock foot. The streamwise extent of the active control surface extends from $(x_c - x_{\text{imp}}) / \delta_m = -8.00$ to $(x_c + x_{\text{imp}}) / \delta_m = 4.00$, where $x_c$ and $x_{\text{imp}}$ denote the control surface leading- and trailing-edge locations in the streamwise direction. Thus, in dimensional units, the actual control surface is $a = 75.6 \text{ mm}$ ($12\delta_m$) long in the streamwise direction and $b = 37.8 \text{ mm}$ ($6\delta_m$) wide in the spanwise direction. As noted earlier, this surface is assumed to have the properties of maraging steel AISI grade 18Ni (300), which finds common use in aerospace applications. The material properties are tabulated in Table 2, where $m$, $\lambda$, and $\nu$ are, respectively, the mass ratio, nondimensional dynamic pressure (representative of the relative structural stiffness), and Poisson ratio. The mass ratio is $m = (\rho_\infty a)/(\rho_i h)$, where $\rho_i$ and $h$ are the material density and thickness, respectively; whereas the nondimensional dynamic pressure is $\lambda = \rho_\infty a^2/D$, where $D$ is the material flexural stiffness. The thickness of the control surface is $h = 0.6 \text{ mm}$ ($h/\delta_m = 0.096$; $h/a = 0.008$); thus, conventional shell elements are used to perform structural failure analysis. The control surface is discretized into 91 × 61 nodes in the streamwise and spanwise directions, respectively, which leads to a total number of 10,800 elements. The leading and trailing edges of the control surface are clamped, whereas the other two edges (side edges) are treated as symmetry boundary conditions for consistency with the flow boundary conditions.

### IV. Results and Discussion

#### A. Features of Baseline Flowfield

The suitability of the inflow profile was first established by comparison with the available experimental data using a Rayleigh-scattering-based nanoparticle-based planar laser-scattering (NPLS) technique [65], which uses Rayleigh scattering, providing high-resolution access to the boundary layer, even near the wall. Bo et al. [50] compare a dimensionless luminescence profile with the theoretical density profile, which is obtained using the adiabatic Crocco–Busemann formula (equation 1.1 of Bo et al. [50]) and the ideal gas law. The theoretical density profile of the incoming turbulent boundary layer and corresponding experimental data obtained from the PIV are shown in Fig. 4a by using a purple line and orange squares, respectively. Figure 4a also incorporates the DNS density profile (solid cyan line) as well as the LES density profile (solid cyan line) at $(x - x_{\text{imp}})/\delta_m = -21.25$ (or at $x = 0$). All profiles exhibit generally good agreement with each other, although experimental and theoretical density values are modestly higher than those from DNS and LES (Fig. 4a).

The time-averaged streamwise velocity and Reynolds stresses are obtained in the LES after an equilibrium turbulent boundary layer is reestablished. The profiles are compared with the corresponding DNS profiles that were used for the digital filter in Figs. 4b and 4c, respectively. The velocity and Reynolds stresses are presented in wall units by scaling the profiles with the wall viscous length scale $\ell_v = U_0/\nu_i$ and the friction velocity of $U_i = (\tau_w/\rho_i)^{1/2}$. Here, $\tau_w$ and $\nu_i$ are the wall shear stress and kinematic viscosity (the subscript $w$ denotes quantities evaluated at the wall). In the LES, the streamwise velocity and Reynolds stresses are also obtained at the $x$ location $(x - x_{\text{imp}})/\delta_m = -21.25$, where the friction Reynolds number is $Re_{\delta} = 410$. The LES profiles of Figs. 4b and 4c show a good match with the DNS profiles that were used for the digital filtering. Thus, the adjustment length in order to establish the inflow turbulence is $\approx 7.5\delta_m$.

The experimental and simulated flow structures in the incoming turbulent boundary layer are displayed in Figs. 5a and 5b, respectively. The former is the PIV image, whereas the latter uses temperature as a surrogate. In general, the organization of flow structures in the close vicinity of the wall as well as in the boundary layer are similar. In particular, the inclination of the evolving flow structures displays an angle of $\approx 42$ degrees to the horizontal, which is nearly identical for both the experiment and the LES inflow boundary layers.

A full 3-D instantaneous flowfield snapshot is shown in Fig. 6 (left frame), where the turbulent boundary layer is displayed using a $Q$ criterion iso-surfaces ($Q = 1$) colored with streamwise velocity. The incident and reflected shocks are shown on a vertical plane at a spanwise extremum with the magnitude of the density gradient $|\nabla \rho|$. The flow deflection of $\alpha = 10.5$ degrees due to the shock generator results in a shock angle of 30.29 degrees and a pressure ratio of $p_3/p_1 = 3$. The expansion waves emanating from the top surface of the trailing edge of the inclination interact weakly with the reflected shock, deflecting it downward toward the bottom surface/outlet as observed in Fig. 6.

A closer view of the separation region is shown in the right frame, where the increased boundary-layer thickness and hairpin-like flow structures are evident. The former effect is due to the adverse pressure gradient associated with the incident shock. The observed features follow those noted in the classical literature [66]. In the boundary layer, the shock manifests as a series of compression waves, forming the foot of the reflected shock upstream of the shock impingement location (Fig. 6). Furthermore, the flow in the near-wall region beneath the SWBLI exhibits negative values of the streamwise velocity component, indicating flow reversal and formation of the separation bubble.

#### B. Effect of Control on Flow Separation

An instantaneous flowfield, displaying the turbulent SWBLI, is captured in the experiments [50] by using the NPLS (Fig. 7a), where the turbulent boundary layer is observed to become thicker due to the adverse pressure gradient imposed by the incident shock. The uncontrolled LES flowfield near the shock impingement location is shown in Fig. 7b using the temperature field. The incident and reflected shocks are evident in both frames for reference. Similar to the experiments, the thickness of the incoming boundary layer increases by $\approx 30\%$ at the shock impingement location.

Figure 7c shows results from the controlled case. Here, the control surface morphs under the action of the algorithm of Sec. II.B by responding to instantaneous flow separation. The control surface progressively deforms for $\approx 1$ flowthrough time, when it reaches an effectively stationary state, and responds minimally to the control

| Table 2 Properties of control surface material: steel |
|---------------------------------|-----------------|
| Parameter          | Value          |
| $\sigma$         | 2035 MPa       |
| $m_r$            | 0.001916       |
| $\lambda$       | 4.93           |
| $\nu$           | 0.30           |
| $h/a$           | 0.008          |
| $b/a$           | 0.5            |
| Nodes ($n_x \times n_y$) | 91 × 61        |
| Elements         | 10,800         |
scheme, due to a smaller flow separation region. The smooth profile of the morphing control surface, even during its deformation toward an asymptotic state (Fig. 7c), is a result of the spanwise averaging and streamwise polynomial smoothing procedures discussed in Sec. II.B. The increase of the boundary-layer thickness due to the shock impingement is much smaller in the controlled case compared to the uncontrolled case (Fig. 7c). Furthermore, active control leads to smearing and shifting of the reflected shock downstream when compared to the uncontrolled experimental and LES results. These effects are further elaborated on in the following:

The separated flow region is visualized with the time-averaged streamwise velocity in Figs. 7d–7f for the experiment (PIV measurements), uncontrolled LES, and controlled LES cases, respectively. The figures identify three isolines of the normalized streamwise velocity ($u/u_\infty$) corresponding to 0.0, 0.1, and 0.5 respectively. The first of these provides an indication of the time-mean reversed...
flow region ($\langle u \rangle < 0$). These plots indicate that the extent of the flow separation for the uncontrolled LES case is $\approx 22$ mm (from $x \approx 234$ mm to $x \approx 256$ mm), as opposed to $\approx 20$ mm in the experiments (from $x \approx 236$ mm to $x \approx 256$ mm). The controlled LES case results in a significant reduction in the size of the separation region, in the mean sense.

A more accurate metric for flow separation is obtained from the time-averaged skin-friction coefficient:

$$C_f = \frac{\tau_w}{(1/2)\rho_w u_w^2} \quad \text{with} \quad \tau_w = \mu \left. \frac{du}{dy} \right|_w$$

where $\tau_w$ is the local skin shear stress on the wall. The skin-friction coefficient and wall pressure profiles across the shock impingement location for the controlled and uncontrolled LES cases are shown in Fig. 8. The negative values of the skin-friction coefficient indicate flow recirculation. For the uncontrolled case, the separation and reattachment points are located at $(x - x_{\text{imp}})/\delta_{in} \approx -4$ and $(x - x_{\text{imp}})/\delta_{in} \approx 0$, respectively; the separated streamwise extent of $\approx 4\delta_{in}$ (Fig. 8a; uncontrolled (NC)) agrees with the $3.51\delta_{in}$ obtained through LES of the same configuration by Grébert et al. [67]. Active surface morphing results in a substantially reduced size of reversed flow, by approximately 50%, in the mean sense (Fig. 8a; controlled (AC)); in this case, the flow separates much farther downstream $(x - x_{\text{imp}})/\delta_{in} \approx -2$ relative to the uncontrolled case, and it reattaches at $(x - x_{\text{imp}})/\delta_{in} \approx 0$. The skin-friction values remain larger for the controlled case, particularly in the region $-5 \leq (x - x_{\text{imp}})/\delta_{in} \leq -2$ when compared to the baseline case, which is already separated here. Figure 8b shows the wall pressure profiles, normalized by freestream pressure, for both cases. The wall pressure for the controlled case differs considerably in the upstream region of $-8 \leq (x - x_{\text{imp}})/\delta_{in} \leq 0$, where it takes higher values for $-8 \leq (x - x_{\text{imp}})/\delta_{in} \leq -4$ and much lower values for $-4 \leq (x - x_{\text{imp}})/\delta_{in} \leq 0$ when compared to the uncontrolled case. In the downstream region of the shock impingement location, the controlled skin-friction and wall pressure values recover to those associated with uncontrolled case.
The spanwise organization of the flowfield for the three cases (experiment, uncontrolled LES, and controlled LES) are compared in Fig. 9 by plotting instantaneous and time-averaged streamwise velocities in an \( xz \) plane at \( y = 0.238\delta_\text{a} \) (\( y = 1.5 \text{ mm} \)) and time-averaged values at \( y = 0.476\delta_\text{a} \) (\( y = 3.0 \text{ mm} \)). The LES spanwise \( z \) extent is 40 mm as opposed to the 80 mm used for the PIV measurements in the experiment. The time-mean streamwise PIV profiles in frames Figs. 9b and 9c show some evidence of spanwise variations. As stated in Ref. [50], such variations in the PIV time-mean profiles are considered to be due to the three-dimensional features induced by the sidewall, noting a similarity with the surface oil flow visualization are considered to be due to the three-dimensional features induced by the sidewall, noting a similarity with the surface oil flow visualization by Babinsky et al. [68]. The uncontrolled LES profiles (Figs. 9d–9f) are obtained for the shorter spanwise extent and spanwise periodic boundary condition, and they do not consider corner effects. The instantaneous LES profile (Fig. 9d) displays very similar features as in the experiment, although more granularity because of the finer resolution in the simulations. The mean profiles at the two heights (Figs. 9e and 9f) also show generally similar reversed velocity regions. Some spanwise undulations are evident in these profiles; similar observations of the reflected shock foot and separation regions have been reported by Grébert et al. [67] for the same configuration, where the authors performed dynamic mode decomposition of their LES flowfields in order to explore the three-dimensionality. On the other hand, the controlled LES case displays much lower negative streamwise velocities (Figs. 9g–9i) and exhibits more uniform spanwise profiles when compared to the uncontrolled experimental and LES results. The three-dimensionality and unsteady characteristics of the baseline and controlled turbulent SWBLIs are explored in the following sections by means of proper orthogonal decomposition and wall pressure power spectral density.

C. Turbulence Flow Structures and Spectra

A comprehensive comparison of the effect of control is greatly facilitated by the use of 3-D decomposition procedures that consider the entire flowfield. In the present case, the important three-dimensional flow structures in the SWBLI can be straightforwardly identified by performing proper orthogonal decomposition (POD) of the full 3-D LES flowfields. The POD method has been extensively used to understand coherent flow structures [69] to model interscale energy exchange [70], as well as to develop reduced-order models [71,72]. For a given number of modes, the POD expansion is optimal in extracting the energy dominant flow structures [73,74]. The snapshot POD approach [75] is employed here since it is computationally much cheaper than the original direct method of POD. The snapshot POD collects instances of the solution vector; these are usually successive time instants of the flow but can, for certain uses, be successive spatial planes [19].

To extract 3-D coherent flow structures in the SWBLI, time instants of the full 3-D flow solution are considered here. A total of \( N_t = 2000 \) snapshots with a time spacing of 100\( \Delta t/\Delta t_\text{a} \) are assembled, by using a suitable inner product, into a cross-correlation matrix, which ensures the capture of low-/high-frequency dynamics. The eigenvalues and eigenmodes of the correlation matrix are then distilled by solving an eigenvalue problem to yield the desired proper orthogonal decomposition. Briefly, if \( s(x, y, z, t) \) represents a time snapshot, where \( x, y, z, \) and \( t \) are the space and time coordinates, the POD of \( s \) is obtained through

\[
\Phi(x, y, z, t) = \Phi_n(x, y, z)\Psi_n(t)
\]

where \( \Phi_n(x, y, z) \) is the \( n \)-th orthonormal space mode, and \( \Psi_n(t) \) is the corresponding orthonormal time coefficient. \( \Lambda_n \) is the eigenvalue associated with the POD modes, which is real and positive.

The first three energy-dominant spatial POD modes of the streamwise velocity for the uncontrolled and controlled cases are displayed in Figs. 10a and 10b, respectively. The three-dimensionality of the POD modes (\( \Phi_1, \Phi_2, \Phi_3 \) in the uncontrolled base case is evident in Fig. 10a. The observation of an alternating pattern in the spanwise

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![Fig. 9 Instantaneous (left column) and averaged (middle and right columns) streamwise velocities over the SWBLI at different wall-normal heights from the experiment (top row) and LES (middle and bottom rows).](http://arc.aiaa.org)
direction is in accordance with the preceding discussion on the spanwise organization of the flow structures, where the LES results indicate spanwise undulation of the streamwise velocity in the horizontal planes (Figs. 9d–9f). In the controlled case, where the surface morphing leads to a considerable (~50%) reduction of the separation bubble, the energy-dominant POD modes appear to be slightly elongated (in the streamwise direction) and smaller in size (in the spanwise direction), as shown Fig. 10b, when compared to the uncontrolled case (Fig. 10a). Thus, the large-scale flow structures of the uncontrolled SWBLI are altered to relatively small-scale flow structures in the controlled case.

The corresponding temporal POD coefficients ($\Psi_u^1$, $\Psi_u^2$, and $\Psi_u^3$) are displayed in Figs. 11a and 11b for the uncontrolled and controlled cases, respectively. As noted before, the temporal POD modes [$\Psi^i(t)$] are also orthonormal, similar to the spatial modes [$\Phi^i(x, y, z)$]. The prominent temporal modes of the uncontrolled case (Fig. 11a) exhibit a relatively lower-frequency response when compared to the controlled case (Fig. 11b). Thus, control has the desired effect of reducing the larger scales and lower frequencies present in the uncontrolled case. The POD modal energies for both cases, normalized using the freestream velocity, are shown in Fig. 11c; the modal energies in the controlled case are seen to be as low as ~50% of the uncontrolled case, particularly for the low-rank POD modes ($n \leq 10$). Clearly, the alteration of the large-scale flow structures leads to a lower total turbulent kinetic energy in the domain, which is simply the sum of all eigenvalues. The mitigation of the low-frequency unsteadiness is also accompanied by a modest increase in mid/high range frequency unsteadiness, which is less structurally sensitive.

To further elucidate the effect of surface morphing on the flow turbulence, the evolution of turbulent kinetic energy (TKE) is tracked...
over the control surface along the streamwise direction. The time-averaged TKE in the full 3-D domain in terms of the POD modes can be expressed as

\[
TKE(x, y, z) = \frac{1}{2} \left( u_i^2(x, y, z, t) \right)
\]

\[
= \frac{1}{2} \sum_{n=1}^{N} \Lambda_n \Phi_n^2(x, y, z)
\]

where \( i \) is the summation index. The wall-normal TKE profiles along different streamwise locations are displayed in Fig. 12a for the uncontrolled case and Fig. 12b for the controlled case. Although, the TKE is normalized by the square of freestream velocity, it is scaled by factor 50 for clarity. The concave surface curvature at the upstream region of the control surface of \(-8 \leq (x - x_{\text{imp}}) / \delta_\text{in} \leq -4\) leads to some enhancement of the TKE compared to the flat surface (Fig. 12b), at about \( (x - x_{\text{imp}}) / \delta_\text{in} \approx -2 \). However, the trend reverses downstream over the entire control surface of \(-4 \leq (x - x_{\text{imp}}) / \delta_\text{in} \leq -2\), where the TKE is substantially suppressed due to the morphed surface (Fig. 12b) as opposed to the uncontrolled SWBLI case (Fig. 12a). In general, the surface convexity is known to have stabilizing effects on the supersonic turbulent boundary layers [78]. The convex region of the morphed surface (Fig. 12b), at about \( (x - x_{\text{imp}}) / \delta_\text{in} \approx -2\), appears to act favorably to reduce the increased levels of TKE in the uncontrolled SWBLI (Fig. 12a). Further downstream of the shock impingement location, the boundary layer undergoes an expansion; in this region \( (x - x_{\text{imp}}) / \delta_\text{in} > 0 \), the controlled SWBLI exhibits marginally lower TKE compared to the uncontrolled case. Overall, the active surface morphing control assists in reducing the turbulence level relative to the uncontrolled case.

The modulation of turbulence due to control may be further examined through the wave number content of wall pressure. In particular, the footprints of the spanwise turbulent structures on the surface can be obtained in terms of the wall pressure power spectral density (PSD) as a function of the spanwise wave number. The premultiplied wall pressure spectra for the uncontrolled and controlled cases along different streamwise locations over the SWBLI region are shown in Fig. 13. The power spectral density \( E_{pp} \) and the spanwise wave number \( \omega_c \) are normalized by the freestream pressure \( (p_\infty = \rho_\infty u_\infty^2) \) and inflow boundary-layer thickness \( \delta_\text{in} \), respectively. Similar to the slight enhancement of the TKE in the upstream concave region of the morphed surface, the level of premultiplied wall pressure PSD is higher \( (x - x_{\text{imp}}) / \delta_\text{in} \approx -6 \) for the controlled case compared to the uncontrolled case, as shown in Fig. 13a.

The flat region of the premultiplied spectra exhibits \( \omega_c^{-4} \) power law for the dependence of the wall pressure PSD \( E_{pp} \) on the wave number \( \omega_c \). In this range of wave numbers \( (E_{pp} \propto \omega_c^{-4}) \), the wall pressure is mainly exerted due to the large eddies of the log region of the turbulent boundary layers that are in equilibrium [79]. In addition to the increased levels of wall pressure spectra due to the SWBLI, the uncontrolled case results in a wider range of the wave numbers exhibiting the \( \omega_c^{-4} \) power law, particularly at the streamwise location of \( (x - x_{\text{imp}}) / \delta_\text{in} = -3.5 \) (Fig. 13b), where the wall pressure spectrum for lower wave numbers \( (\omega_c \lesssim 4) \) also tends to exhibit the \( \omega_c^{-4} \) dependence. As reported in Ref. [80], the wall pressure spectra at the lower wave numbers display \( \omega_c^0 \) dependence, i.e., \( \omega_c \) dependence for the premultiplied spectra. The uncontrolled SWBLI deviates from the \( E_{pp} \propto \omega_c^0 \) power law (i.e., \( \omega_c E_{pp} \propto \omega_c^1 \)) for the streamwise locations that fall inside the flow separation region, as shown in Figs. 13b and 13c. On the contrary, the controlled case exhibits \( \omega_c E_{pp} \propto \omega_c \) dependence for lower wave numbers at all locations (Fig. 13). Although the SWBLIs for both the controlled and uncontrolled cases lead to increased levels of wall pressure spectra, the surface morphing inhibits the increase of PSD, levels particularly for the lower wave numbers and inside the separated flow region. Thus, we can anticipate a favorable effect of the surface morphing for reducing the low-frequency unsteadiness.

As elaborated on in Sec. I, although the flow separation is the direct measure employed for the control strategy, a key feature of the SWBLI under consideration is the low-frequency unsteadiness of the separation bubble and its impact on the structure. The wall pressure frequency spectra along the centerline \( (z = 0) \) for the uncontrolled and controlled SWBLI cases are displayed in Fig. 14. The uncontrolled case (Fig. 14a) clearly exhibits a low-frequency peak near the flow separation location of \( (x - x_{\text{imp}}) / \delta_\text{in} \approx -4 \). The Strouhal number of this frequency peak based on the freestream velocity and inflow boundary-layer thickness is \( St = f \delta_\text{in} / u_\infty \approx 0.005 \). The value is \( \approx 0.02 \) based on the flow separation length of \( \approx 4 \delta_\text{in} \). These Strouhal number values are typical for the low-frequency unsteadiness of SWBLIs discussed in the literature [5,12]. In addition, the SWBLI
results in midrange frequencies of $0.01 \leq St \leq 0.1$ as well as a higher magnitude of PSD at turbulent frequencies ($0.1 \leq St$); the details of these have been discussed by Adler and Gaitonde [7].

The surface morphing control, on the other hand, leads to a considerable mitigation of the wall pressure spectral energy, particularly for the low-frequency unsteadiness. The decrease of the peak spectral energy is about $\approx 10\%$ for the controlled case as compared to the baseline case. The wall pressure PSDs at two probe locations of $(x - x_{imp})/\delta_{in} = -6$ and $(x - x_{imp})/\delta_{in} = 0$ are displayed in Fig. 14 for the uncontrolled and controlled cases. The effect of surface morphing control on the low-frequency unsteadiness is also evident in the wall pressure spectra. For the controlled case, the wall pressure PSD at the shock impingement location shows two distinct peaks at Strouhal numbers of $St \approx 0.1$ and $St \approx 1$, which correspond to the mid- and high range frequencies, respectively. This is consistent with the aforementioned POD analysis, where we observe the mitigation of large-scale low-frequency flow structures accompanied by a modest increase of the mid-/high-frequency unsteadiness due to the control. In addition, as noted before, the controlled SWBLI results in a substantial reduction ($\approx 50\%$) of the flow separation in the mean sense. The wall pressure spectra for the controlled case exhibits, in general, a favorable effect of the control on the low-frequency unsteadiness.

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**Fig. 14** Wall pressure frequency spectra at the midspan (spanwise averaged) for a) uncontrolled and b) controlled cases.

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**Fig. 13** Spanwise wall pressure wave number spectra at different streamwise locations for the uncontrolled and controlled SWBLIs.
D. Structural Integrity of the Control Surface

The control technique is initiated after the turbulent SWBLI is fully established, which occurs at a flow development time of $tu_a/δ_{in} = 600$. However, as discussed by Shinde et al. [49] in the context of transitional SWBLI control, surface morphing can be activated during the transients, before the impingement of incident shock; depending on how the flow is established; this may be useful to avoid the initiation of the flow transition. On the other hand, for the present turbulent SWBLI, it can be used to hinder the full establishment of the separation bubble or interfere with the establishment of structurally detrimental low-frequency dynamics.

In this section, the transient response is examined further. The response depends on the material chosen for the morphing surface. Our earlier work [49] evaluated the behavior of two materials for the control surface, namely, aluminum alloy AI-7075-T6 and maraging steel AISI grade 18Ni (300). As noted earlier, the choice of the latter in the present work is motivated by the lower von Mises stress values relative to yield stress for the same structural deformation.

The time evolution of the structural deformation at the center of the control surface and the induced von Mises stress are displayed in Fig. 15a. The control surface morphs and reaches its final deformation control surface and the induced von Mises stress are displayed in relative to yield stress for the same structural deformation. The structural stress rises steeply during this time, and it eventually saturates to $σ/σ_y ∼ 0.8$. The asymptotic profile of the control surface deformation is shown in Fig. 15b for $tu_a/δ_{in} ≈ 40$, where the control surface responds only minimally to the instantaneous flow separation. The corresponding von Mises stress profile is shown in Fig. 15c, where the peak values of the stress occur near the clamped edges of the control surface, particularly near $(x − x_{imp})/δ_{in} ≈ 2$ the downstream edge of the control surface.

V. Conclusions

An active control technique, in which the surface beneath a turbulent separated region is deformed through morphing, is examined with the goal of reducing separation and accommodating structurally detrimental unsteadiness. An impinging shock-wave/turbulent boundary-layer interaction is considered, with the shock strength sufficiently large to trigger separation of the boundary layer at the chosen Mach (2.7) and Reynolds numbers. The morphing-enabled patch is placed so that it straddles the uncontrolled separation region. The adaptive deformation, subject to material constraints ensuring elastic behavior during the transient state, is specified based on a local separation criterion using the reversed velocity magnitude, which is proportional to the streamwise surface shear force component. A large-eddy simulation is employed to resolve the separation and unsteadiness details, as well as to provide an understanding of the underlying fluid dynamics under the influence of this type of control. The uncontrolled baseline LES shows good agreement with the experimental results in terms of flow separation and flow structure organization in the SWBLI. The surface morphing simulation yields an asymptotic state comprising a protrusion with a height of $0.32δ_{in}$, with the peak located $2.25δ_{in}$ downstream of the uncontrolled separation point. A primary effect on pressure is the reduction of the initial uncontrolled pressure rise, which delays separation initiation, while keeping the overall rise the same as the uncontrolled value. The effect is to reduce the size of the separation bubble substantially (by about 50%). Analysis of the power spectral density of the wall pressure indicates that the major regions of low-frequency events are pushed downstream and are of smaller amplitude. Each of the three most energetic structures obtained with POD indicate a reduction in energy and a diminution of structure size. Morphing thus facilitates the evolution of an effective bump that can suitably adjust itself to changing local conditions. Future work should focus on the optimization of different parameters, such as relaxation constants and streamwise control surface size, to yield the smallest separation region and largest reduction in low-frequency energy content. The influence of fluid–structural coupling with the final morphed bump is also a potential area worthy of examination.

Appendix: Nonlinear Least-Squares Using Gauss–Newton Algorithm

The Gauss–Newton algorithm to solve nonlinear least-squares problems is a modification of Newton's method for finding a minimum of a function and may be summarized as follows. In contrast to Newton’s method, the Gauss–Newton algorithm is limited to mini-
mization of a sum of squared function values but does not require computation of the second derivatives, which can be difficult to obtain. The data-fitting objective is to find a set of parameters $\kappa$ such that a model function $y_f(x, \kappa)$ results in a minimum error/residual $R_f$ for data points $(x_i, y_i)$, where $x_i$ and $y_i$ are the streamwise $x$ location and deflection of the control panel. A model function polynomial of order $N-1$ of form

$$y_c = \sum_{n=1}^{N} \kappa_n x_i^{n-1}$$  \hspace{1cm} (A1)

is considered for the deformation of the control surface. $N = 13$ is used in the present work, resulting in a polynomial function of the 12th order. The residuals $R_f$ are expressed as

$$R_f(\kappa) = y_{ci} - f(x_{ci}, \kappa)$$  \hspace{1cm} (A2)

A Jacobian matrix for the residuals with respect to parameter $x$ is given as

$$J_{ij} = \frac{\partial R_f(\kappa)}{\partial x_j}$$  \hspace{1cm} (A3)

The minimization proceeds with an initial guess for the parameters $\kappa^{(0)}$, and the Gauss–Newton method can be expressed as

$$\kappa^{(k+1)} = \kappa^{(k)} + (J^T J)^{-1} J^T R$$  \hspace{1cm} (A4)

where $(J^T J)^{-1} J^T$ is the pseudoinverse of $J$, which is obtained with Doolittle Lower–Upper (LU) factorization.

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