A theoretical model of fluidelastic instability in tube arrays

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ABSTRACT

A theoretical model of the fluidelastic instability in tube arrays is presented in this article. It is developed for a normal-square cylinder array and then extended to other types of array patterns. The model is based on transient interactions between a single cylinder and the adjacent flow streams of single phase fluid. The central cylinder is assumed to oscillate as a one-degree-of-freedom mass on a spring system in the lift direction only. A small displacement of cylinder is assumed to perturb the surrounding interstitial flow, while as for higher displacements the cylinder causes flow distortions in regular intervals. These disturbances are convected downstream along with the interstitial flow. The waveforms of these flow distortions are assumed to interact with the array pattern, thence modifying the fluid force acting on the cylinder. The critical flow velocity is obtained as a function of mass ratio and damping parameter. The proportionality constant of the mathematical model is derived in terms of the pitch ratio and Euler number. The mathematical development results in an implicit model for the critical flow velocity. The model predictions are in a good agreement with experimental results.

1. Introduction

The flow-induced vibrations in the heat exchanger tube arrays exhibit different mechanisms. The vibrations are generally classified under, vortex-induced vibration, turbulent buffeting, acoustic vibration and the fluidelastic vibration. The underlying mechanisms in the first three types of vibration are well understood. The safe operating conditions can be procured against these vibration types by appropriate design guidelines. The exact mechanism underlying the fluidelastic instability is relatively less understood. The damage due to the fluidelastic instability is generally severe and occurs within relatively short time. The fluidelastic instability is extensively studied in order to accurately understand and predict the critical velocity thresholds. The presence of fluidelastic excitations in the context of cylinders was first reported in Roberts (1962). The work of Connors (1970) and Connors, 1978 led to a simplified model for the fluidelastic instability,

\[ \frac{\nu_{pc}}{f_{pc} D} = K \left( \frac{m_d}{\rho D^2} \right)^a \]  

(1)

where, \( \nu_{pc}, f_{pc} \) and \( D \) are the critical pitch (minimum gap) velocity, natural frequency and the diameter of the cylinder respectively. The non-dimensional critical pitch velocity is proportional to the mass \( m \), logarithmic decrement \( \delta \) of the cylinder vibration in the non-dimension forms with the exponent \( a \). \( K \) is the constant of proportionality. \( \rho \) is the fluid density. An enormous amount of work is carried out in terms of experiments and theoretical models, since the work of Connors (1970), in order to better understand and predict the phenomenon. The topic is well reviewed in Paidoussis (1983), Weaver and Fitzpatrick (1988), Pettigrew and Taylor (1991) and more recently in Paidoussis et al. (2010, Chapter 5). A detailed review on the mathematical models of fluidelastic instability is provided in Price (1995).

In this article, a new mathematical model for the fluidelastic instability is presented. It is based on dynamic interactions between a single cylinder and its adjacent fluid flow. The flow perturbations due to the cylinder motion are modeled as waveforms on top of the interstitial fluid flow. The flow streams carrying these perturbations interact elastically with the cylinder, especially for the low mass ratio \( (m/\rho D^2) \). The mathematical development and a procedure to estimate the critical pitch velocity \( \nu_{pc} \) is formulated in the following sections. The model predictions are compared with a set of experimental data listed in Pettigrew and Taylor (1991) as well as with a large experimental data reported in Paidoussis et al. (2010, Chapter 5)) for all the four array patterns.

2. Theory

The cross flow in normal-square tube arrays forms a typical flow pattern, which consists flow channels with varying cross-sectional area.
The interstitial flow velocity varies depending on the cross-sectional area. The flow accelerates between adjacent cylinders of a row (lower cross-sectional area), while as it decelerates between the two rows of cylinders (larger cross-sectional area). A motion of cylinder in the flow normal (or lift) direction results in a decrease in the cross-sectional area of an adjacent flow channel on one side of the cylinder, and at the same time an increase in the cross-sectional area on the other side of the cylinder. Consequently, the local flow velocity either increases or decreases accordingly. These perturbations are conveyed away from the cylinder. The mathematical model proposed in the following section is based on these dynamic interactions between the flow streams and a cylinder of the normal-square (90°) array.

2.1. Mathematical model

The kernel of a normal-square (90°) array is shown in Fig. 1. The diameter and pitch distances are represented by D and P respectively. The pitch ratio (p° = P/D) is the same in both the longitudinal (in-flow) and transverse (flow-normal) directions. The inflow direction is shown by the bold arrows. The central cylinder is assumed to oscillate in the direction of the lift force only, designated here as flow normal direction. The schematic physical representation of the mass on a spring is shown in Fig. 1, where k, c stand for the cylinder stiffness and damping respectively. The mass per unit length of the cylinder is represented by m. The mass includes the hydrodynamic mass of the fluid medium at rest. Similarly the stiffness (k) and damping (c) coefficients are defined with respect to the quiescent fluid medium. Eq. (2) represents the motion of the cylinder in the flow normal direction. y is the instantaneous displacement of cylinder in this direction. t represents the time.

The right hand term of the equation is a sinusoidal fluid force with an amplitude \( f_0 \) per unit length of the cylinder and an angular periodicity (\( \omega_h \)) associated with the force.

\[
\frac{d^2y}{dt^2} + \frac{c}{m} \frac{dy}{dt} + ky = f_0 e^{-\imath \omega_h t} \tag{2}
\]

where \( \imath = \sqrt{-1} \). Using the definitions of the natural angular frequency (\( \omega_h \)) and damping ratio (\( \zeta \)) of the cylinder, \( \omega_h = \sqrt{k/m} \) and \( \zeta = c/2\sqrt{km} \), Eq. (2) can be written as,

\[
dy^2 \frac{dt}{dt^2} + 2\zeta \omega_y \frac{dy}{dt} + \omega_y^2 y = \frac{f_0}{m} e^{-\imath \omega_h t} \tag{3}
\]

The general solution can be given as,

\[
y = Y e^{-\imath (\omega_h + \theta)} \tag{4}
\]

where, Y is the magnitude of cylinder oscillations, while as \( \theta \) is the phase difference between the fluid force and the cylinder response (y). The magnitude (Y) can be obtained by solving Eqs. (4) and (3).

\[
-\sqrt{\omega_h^2 + 2\zeta \omega_h \omega + \omega_y^2} = \frac{f_0}{m} \cos(\theta)
\]

\[
-2\omega_h \omega + \omega_y^2 = \frac{f_0}{m} \sin(\theta)
\]

Thus,

\[
Y = \frac{f_0}{\sqrt{\omega_h^2 + \omega_y^2}} \tag{5}
\]

\[
\begin{align*}
\langle \omega_h^2 - \omega_y^2 \rangle & = \frac{f_0}{m} \cos(\theta) \\
\langle -2\omega_h \omega \rangle & = \frac{f_0}{m} \sin(\theta)
\end{align*}
\]

The unsteady response amplitude (Y) of the cylinder is directly proportional to the magnitude of the fluid force \( f_0 \) and it is inversely proportional to the mass and damping terms. The phase difference (\( \theta \)) between the fluid force acting on the cylinder and cylinder displacement is considered as an important component of the fluidelastic instability, particularly in the theoretical models based on Lever and Weaver (1982). The exact physics of the phase lag (\( \theta \)) is not well understood (Khalifa et al., 2013). The phase lag is approximated by using an expression based on a hydraulic analogy in Lever and Weaver (1982). In Eq. (7), the phase lag (\( \theta \)) is eliminated in the derivation of the displacement amplitude (Y), although its effect is incorporated in the square-root term.

The fluid force (\( f_0 \)) in Eq. (7) can be expressed in terms of the pitch velocity (\( u_h \)) by an empirical relation as,

\[
f_0 = E u \frac{1}{2} D \rho u^2 \tag{8}
\]

where, \( E u \) is an instantaneous component of the Euler number in the transverse direction. The Euler number in heat exchanger designs is commonly defined as,

\[
E u = \frac{\langle \Delta p_{row} \rangle}{2 \rho u^2} \tag{9}
\]

where, \( \Delta p_{row} \) is an instantaneous pressure drop across a row of an array and \( \langle \cdot \rangle \) represents ensemble averaging operation. An instantaneous Euler number in the flow direction can be given as,

\[
E u_k = \frac{\Delta p_{row}}{2 \rho u^2} \tag{10}
\]

Similarly, the flow normal component of the Euler number \( E u_k \) is assumed to be based on the instantaneous pressure drop in the lift direction, \( \Delta p_{row} \), across the cylinder. By using Eq. (8) in Eq. (7),

\[
Y = \frac{E u_k \frac{1}{2} D \rho u^2}{m (\frac{1}{1 - \frac{\eta}{\eta_m}})^2 + (2 \frac{\eta}{\eta_m})^2} \tag{11}
\]

The term in the square root acts as a mechanical impedance, which signifies the resistivity of the cylinder to the imposed harmonic force. Let the mechanical impedance be,
\[ L_n = \sqrt{\left(1 - \frac{\omega_h}{\omega_n}\right)^2 + \left(2\frac{\lambda}{\omega_n}\right)^2} \]  

(12)

Rearranging the terms in Eq. (11) gives,

\[ \frac{m}{\rho} Y_{fl} = \left( \frac{E u_y}{2(2\pi)^2} \right) \frac{u_y^2}{f_n^2} \]  

(13)

The amplitude (Y) of the cylinder vibration can be expressed in terms of the fraction (h) of the minimum distance between two cylinders (P−D) as,

\[ \frac{Y}{D} = \frac{h(P-D)}{D} = h(p^* - 1) \]  

(14)

The expression for the pitch velocity (u_y) using Eqs. (13) and (14) becomes,

\[ \frac{u_y}{L_n D} = \sqrt{\frac{8\pi^2 h (p^* - 1)}{E u_y} \left( \frac{m}{\rho D^2} \right)^{0.5} \left( \frac{\lambda}{m} \right)^{0.5}} \]  

(15)

2.1.1. Impedance modeling and stability mechanisms

The impedance \( L_n \) represents the dynamics between the flow periodicity (\( \omega_h \)) and cylinder natural frequency (\( \omega_n \)) at a particular flow velocity (\( u_h \)). It also contains the damping ratio (\( \zeta \)). The dynamic interactions between the flow streams and the cylinder oscillations can be modeled by using the impedance \( L_n \). The oscillating central cylinder perturbs the adjacent flow streams. The perturbations, in terms of the local high/low velocities, travel on top of the flow streams. The perturbations appear on the flow streams in a regular interval, since they are generated as a result of the harmonic oscillations of the cylinder. The distance between the two high/low velocity perturbations (or simply the wavelength of the perturbation wave) \( \lambda \) can be defined as,

\[ \lambda = \frac{2\pi u_y}{\omega_h} \]  

(16)

The flow channels with the flow perturbation waves is idealized in Fig. 2. The flow direction is shown by the bold arrows. The smaller waves with a wavelength \( \lambda \) represent the perturbation waves. Similarly, the pitch length (P) can be expressed in terms of the angular flow frequency (\( \omega_h \)), which represents the flow periodicity due to the array pattern.

\[ P = 2\pi \frac{u_y}{\omega_h} \]  

(17)

The flow perturbations produced due to the cylinder vibration travel with the adjacent flow streams through the non-uniform cross-sectional areas of the array pattern. The wavelength \( \lambda \) increases linearly with the pitch velocity \( u_y \) as per Eq. (16). At a particular value of the velocity (\( u_h \)), where \( \lambda \) equals the pitch distance (\( \lambda \approx P \)), the impedance (\( L_n \)) of the system becomes minimum. Thus the cylinder experiences less resistance for the vibration. The amplitude of oscillations increases as per Eq. (11), in other words, the effective or fluidelastic damping (i.e. the damping of the cylinder \( \zeta \) plus the fluid damping coefficient) decreases. In a contrary scenario, the increase of impedance results in an increase of the effective damping of the cylinder vibration. The increase or decrease of the effective damping of cylinder reflects in the respective decrease or increase of the amplitudes of cylinder vibration (Eq. (11)).

Thus the impedance (\( L_n \)) varies dynamically for uniformly increasing flow velocity (\( u_h \)). In general, we can consider \( n \lambda / P \) for \( \lambda \in P \) and \( \lambda \approx nP \) for \( \lambda \geq P \), where \( n \) is a positive integer multiplier. Let \( \alpha \) be the ratio of the wavelength \( \lambda \) and the pitch distance \( P \) of the array. We can write,

\[ \alpha = \frac{\lambda}{P} = \frac{\omega_h}{\omega_n} \]  

(18)

The fluidelastic instability model presented by Yetisir and Weaver (1993), Yetisir and Weaver (1993) is based on the semi-analytical model of Lever and Weaver (1982). The unsteady model of Lever and Weaver (1982) is improved in Yetisir and Weaver (1993), Yetisir and Weaver (1993) by using a decay function for the perturbations away from an oscillating cylinder, in addition to the phase lag function used in Lever and Weaver (1982). In these formulations the flow disturbances are accounted in terms of the perturbations in the area of the flow channel. Thus the perturbations in the flow channel area, introduced by a moving tube, are assumed to decay away from the tube. A simple decay function for the area perturbation is used (refer Eq. (2) in Yetisir and Weaver (1993)). Similarly, in this formulation, the influence of the perturbation generated by a cylinder is assumed to decay exponentially as it travels away from the cylinder. The ratio \( \alpha \) can be redefined as \( \beta \), for the multiple synchronizations between \( \lambda \) and \( P \) as well as by taking into account the exponential decay as,

\[ \beta = 1 + \sin(\pi \alpha) \exp(-\alpha) \]  

(19)

The impedance \( L_n \) for the tube array can be redefined using \( \beta \) as,

\[ L_n = \sqrt{(1-\beta^2)^2 + (2\beta P)^2} \]  

(20)

The effect of parameter \( \beta \) on the mechanical impedance (Eq. (20)) is shown in Fig. 3. Fig. 3(a) shows the variation in the mechanical impedance and its terms for increasing \( \alpha \), where \( \beta = \alpha \). It corresponds to a forced harmonic oscillator whose response can be categorized based on the value of \( \alpha \). For \( \alpha = 1 \), the first term (\( 1-\beta^2 \)) of the impedance becomes zero and the second term (\( (2\beta P)^2 \)), which contains damping, controls the response of the cylinder, hence it is damping controlled oscillations. Similarly, for \( \alpha \ll 1 \) and \( \alpha \gg 1 \), it is stiffness and mass controlled oscillations respectively. However, because of the presence of adjacent cylinders and the interstitial flow, we redefine \( \beta \) as in Eq. (19). The variations in the impedance and its terms for \( \beta = 1 + \sin(\pi \alpha) \exp(-\alpha) \) are shown in Fig. 3(b). The first term (\( 1-\beta^2 \)) significantly contributes to the impedance for smaller values of \( \alpha \), except when \( \alpha = 1, 2, ... \), for which the damping term contribution becomes dominant. In this range of \( \alpha \), \( 1 \leq \alpha \leq 3 \), stiffness and damping terms compete to supersede each other depending on the synchronization between the perturbation wavelength (\( \lambda \)) and array longitudinal pitch (\( P \)). In an analysis based on experimental data (Tanaka et al., 2002), the stiffness and damping controlled mechanisms were observed to assist each other and the instability comprised a combination of the both mechanisms for the lower range of mass-damping parameter. The interplay between these two mechanisms is attained by the sin term in the definition of \( \beta \) (Eq. (19)).

Fig. 4 shows a qualitative comparison between the effective fluidelastic damping in the pre-instability regime (\( u_h \leq u_{cr} \)), obtained by using the unsteady model of Tanaka and Takahara (1980) (Fig. 4(a)) and
the mechanical impedance for $\alpha < 1$ (Fig. 4b). A similar comparison of the fluidelastic damping between the mathematical models of Lever and Weaver (1986), Price and Paidoussis (1984) and Tanaka and Takahara (1980) is provided in (Paidoussis et al. (2010, Chapter 5)), where Tanaka and Takahara, 1980 prediction of the fluidelastic damping is expected to be accurate since the model is based on measured experimental data. The normalized fluidelastic damping in the pre-instability regime (Fig. 4a), for a low mass ratio ($m/rD^2 = 10$), shows an initial increase followed by a decrease with a maximum at $u_p/u_{pc} \approx 0.5$. The fluidelastic damping decreases to zero at $u_p/u_{pc} = 1$, i.e. at the onset of instability. The term $\sin(\pi \alpha)$ of Eq. (19) assists in exhibiting this behavior of the instability, consequently the impedance increases for $0 \leq \alpha \leq 0.5$ and decreases for $0.5 \leq \alpha \leq 1$ (see Fig. 4b). For $\alpha < 1$ (Fig. 4b), the stiffness term mainly contributes to the mechanical impedance. In this case, the structural damping does not play a very important role in the instability. This is also coherent with the mathematical formulations in Chen (1983), Chen and Jendrzejczyk (1983), Tanaka and Takahara (1981) and Gibert et al. (1977), where the damping term is separated from the mass term and lower values of the exponent are suggested for the damping term.

The perturbations generated due to cylinder oscillations can have only finite span of the spatial correlations, similar to the correlations observed in other turbulent flows (Shinde et al., 2014). Thus for the large values of $\alpha$, $\alpha = 3, 4, \ldots$, the resonance condition becomes fuzzier. The choice of an exponential decay function for modeling the turbulent correlations and coherences is ubiquitous. In the tube arrays configurations, it has been used in several works, such as (Berland et al., 2014; Axsata et al., 1990; Rowe et al., 1974 and Corcos, 1963). Similarly, an exponential decay function $\exp(-\alpha)$ (Eq. (19)) is used in this formulation in order to model the perturbation decay.

At higher values of the mass-damping parameter (typically for gaseous flows), the mechanical impedance ($I_m$) simply becomes $I_m = 2\zeta$. Thus, Eq. (22) takes the form of Eq. (1) with an exponent value $a = 0.5$, which is common for both the mass ratio ($m/rD^2$) and the damping ratio ($\zeta$).

The fluidelastic instability is usually classified under different mechanisms, mainly, in the stiffness controlled and damping controlled mechanisms, as predicted by Chen (1983), Chen (1983) as well as Price (1986). In the damping controlled mechanism the fluidelastic forces are in phase with the cylinder velocity, while in the stiffness controlled mechanism the forces are in phase with the cylinder displacement. At
the low values of mass-damping parameter \((m\delta/\rho D^2)\), the instability is considered as damping controlled, while at the higher mass-damping parameter values the instability is considered as stiffness controlled. Furthermore, in Tanaka et al. (2002) these two mechanisms are found to assist each other for lower values of the mass-damping parameter, and hence the instability is considered to be a combination of both mechanisms. At the higher values of the mass-damping parameter, they observed a change in the vibration pattern, which they attribute to a change of instability mechanism.

The phase lag \((\theta)\) between the cylinder displacement and the forces can be approximated in terms of the modeled mechanical impedance (Eq. (20)) and the parameter \(\beta\) as,

\[
\theta = \arcsin \left( \frac{2\beta P}{Im} \right)
\]  

(21)

**Fig. 5** shows the variation of the phase angle between the cylinder displacement and the forces on it for increasing \(\alpha\) or \(u_p^*/P^*\). It also shows the effect of structural damping \(\zeta\) on the phase angle \(\theta\). For increasing values of \(\zeta\), the phase change, from 0° to 90°, occurs at lower values of \(\alpha\) or \(u_p^*/P^*\). At low values of mass-damping parameter, where the critical velocities are also low, the parameter \(\beta\) oscillates about 1 for the increasing flow velocity. The phase lag \(\theta\) oscillates between 0 and 90deg, which indicates the presence of both stiffness as well as damping controlled mechanisms, an observation similar to the Tanaka et al. (2002). For high values of the mass-damping parameter (or the high critical flow velocities), the parameter \(\beta\) converges to 1 as the flow velocity increases, which leads to the phase difference of \(\theta = \pi/2\) (90deg). Although this implies that the fluid forces are in phase with the cylinder velocity, the higher values of mass-ratio \((m/\rho D^2)\) result into motion dependent forces (Chen, 1983; Chen, 1983; Price, 1986) on the cylinder. Therefore at these values of mass-damping parameter, the instability is expected the stiffness controlled mechanism. An elaborated discussion on the phase lag and stability mechanisms is provided in (Paidousis et al., 2010, chapter 5).

2.1.2. Stability criteria and estimation of the critical flow velocity

The amplitude of the cylinder vibration \((\gamma = h(p^* - 1)D)\) increases (or may also decrease depending on the dynamics between the perturbation waves and the array pattern) with the increasing pitch velocity \((u_p)\), as per Eq. (15). The unsteady and stationary response of the cylinder is expected to follow stable limit cycles, for small amplitude vibrations \((h < 0.5)\). The limiting value of \(h\) is 1 for a single cylinder oscillating in an otherwise rigid normal-square (90°) tube bundle. For \(h > 0.5\), the oscillating cylinder, at its peak displacement, blocks more than 50% cross sectional area of as adjacent flow channel, at the same time, it provides 50% more cross sectional area to the other adjacent flow channel (refer Shinde, 2015, Chapter 3, Fig. 3.14(n)), the cross-flow between the adjacent flow channels is evident in the figure. The response of cylinder is expected to follow unstable limit cycles with a divergence, this is considered as the onset of the instability. Thus the critical value for the amplitude of cylinder oscillations can be taken as half the gap distance \((P-D)\), i.e., \(h_c = 0.5\). In general, the amplitude of cylinder vibration more than 1% of its diameter is considered to be critical in the design guidelines. The following section Section 3 sheds more light on the sensitivity of the critical flow velocity to this parameter.

The Euler number \((Eu_c)\) in the lift direction is defined in terms of the pressure drop \((\Delta P_f)\) across the cylinder in the lift direction. The pressure drop in the lift direction remains small for smaller oscillation amplitudes of cylinder. To authors’ knowledge, the instantaneous or time averaged pressure drop or Euler number in the lift direction across an oscillating cylinder is difficult to gather and it is unavailable. Furthermore, the time averaged value of Euler number in the drag direction changes from \(0.3\) to \(0.28\) for Reynolds number varying from \(10^3\) to \(10^6\) respectively (refer Fig. 6). The variation in Euler number is fairly insensitive to the change in Reynolds number, therefore it is reasonable to assume the time averaged values of the Euler number in lift direction to follow same trends and have values in the same range. At the onset of instability the value of Euler number \((Eu_c)\) can be obtained for the corresponding critical Reynolds number \((Re_c)\) by using Eqs. (24) and (25). Eq. (15) can be written for the critical pitch velocity \((u_{pc})\) as,

\[
\frac{u_{pc}}{f_D} = K\left(\frac{m}{\rho D^2}\right)^0.5 I_{m0.5}^0.5
\]

(22)

where the constant of proportionality \(K_c\) is,

\[
K_c = \sqrt{8\pi^2 h_c (p^* - 1)} = 2\pi P^* - 1
\]

(23)

The derivation of critical pitch velocity \((u_{pc})\) in Eq. (22) is implicit. The terms \(Eu_c\) in the constant of proportionality \((K_c)\) and the wavelength \(\lambda\) in the impedance \(I_m\) are functions of the pitch velocity \(u_{pc}\) itself. The Euler number \((Eu)\) for different Reynolds numbers and array configurations is generally provided in the heat exchanger design handbooks in terms of empirical relations. Eqs. (24) and (25) represent power series providing the values of Euler number \((Eu)\) for different Reynolds numbers \((Re)\) and for the normal-square (90°) and rotated-square arrays (45°). The values of the empirical constants \(c_i\) varies with the array configuration. The table in Fig. 6 provides the values of the empirical coefficients \(c_i\) for the normal-square arrays for different pitch ratios \((p^*)\). Fig. 6 shows the curves generated using these empirical relations.

\[
Eu = \sum_{i=0}^{4} \frac{c_i}{Re^i} \quad \text{normal–square and rotated–square arrays}
\]

(24)

\[
= \sum_{i=0}^{4} c_i Re^i \quad \text{normal–square arrays, longitudinal pitch ratio } p^* = 2.5
\]

(25)

An iterative procedure can be followed in order to solve Eq. (22) for \(u_{pc}\). The structure parameters, namely mass \((m)\), natural frequency \((f_D)\) and damping ratio \((\zeta)\) in a quiescent fluid medium are known a priori. Similarly, the fluid parameters, namely, fluid density \((\rho)\), fluid viscosity \((\mu)\) and the geometrical parameters \((D, P)\) are known beforehand. The critical pitch velocity \(u_{pc}\) is thus estimated using Eq. (22) for an arbitrary value of the pitch velocity \(u_p\), such that \(u_{pc} < u_p\). Fig. 7 shows the flow-chart of the procedure to estimate the critical pitch velocity. On one hand, for an arbitrary value of the pitch velocity \((u_p)\), Reynolds number \(Re\) is calculated using the density \(\rho\), viscosity \(\mu\) and cylinder diameter \(D\). The Euler number is estimated using the value of Reynolds number and an appropriate empirical relation (Eq. (24) or (25)) and the
values of coefficients. The Euler number \((Eu)\) and the critical value of the fraction \((h_c)\) are used to estimate the critical proportionality constant \(K_c\) using Eq. (23). On the other hand, the perturbation wavelength \(\lambda\) is obtained using the arbitrary pitch velocity \(u_p\) and the natural frequency of the cylinder \(f_n\) by using \(\lambda = u_p/f_n\) relation. The parameter \(\alpha\) and \(\beta\) can be readily obtained by using Eq. (18) and (19) respectively. The mechanical impedance \(I_m\) is estimated by using the damping ratio of cylinder \((\zeta)\) and the parameter \(\beta\) in Eq. (20). Thus for an arbitrary pitch velocity \(u_{pc}\), a critical value of the pitch velocity \(u_{pc}\) can be obtained using Eq. (22). The procedure is repeated until \(u_p \geq u_{pc}\).

### 3. Results and discussion

The experimental results of the fluidelastic instability in normal-square (90°) as well as normal-triangular (30°) tube arrays are taken from the review article of Pettigrew and Taylor (1991). The results of 11 experiments and corresponding model predictions for normal-square (90°) are enlisted in Table 1. The remaining experimental data of the Table 1 for normal-triangular (30°) will be discussed in Section 4. In the second column of the table, the names of experiment series are listed, which refer to the corresponding data source. Please refer to Table 2 of Pettigrew and Taylor (1991) for further details. The data set AMOVI is the experimental results of Cardolaccia and Baj (2015), while DIVA are data from CEA (Commissariat à l’Énergie atomique et aux Énergies alternatives) experiments, referenced in Shinde et al. (2014). All the experiments are carried out with the water flows except the fifth one, which is with the air flow. The pitch ratio \((p^*)\) and the cylinder diameter \((D)\) in \(m\) are listed in the third and fourth columns respectively. The cylinder mass per unit length \((m)\), natural frequency \(f_n\) in \(Hz\) and the damping ratio \((\zeta)\) are listed in the fifth, sixth and seventh columns respectively. These quantities are defined with respect to the fluid medium at rest. The fluid density \(\rho\) in \(kg/m^3\) and viscosity \(\nu\) in \(Pa\cdot s\) are tabulated in the eighth and ninth columns of the table. The values of mass-damping parameter \((m\delta/\rho D)\) are listed in the tenth column. The critical pitch velocities \((u_{pc})\) in \(m/s\) for each experiment are provided in the eleventh column of the table. The non-dimensional critical pitch velocities (critical reduced velocities) \((u^*_{pc})\) are listed in the twelfth column. The remaining columns of the table provide the results of model predictions. The critical reduced pitch velocity \((u^*_{pc})\) and the critical pitch velocity \((u_{pc})\) in \(m/s\) are estimated using Eq. (22). They are listed in the thirteenth and fourteenth columns of the table. The corresponding Reynolds number \((Re)\), Euler number \((Eu)\) and the proportionality constant \((K_c)\) are tabulated in the fifteenth, sixteenth and the seventeenth columns of the table respectively.

It is necessary to note that the experimental results compiled in Pettigrew and Taylor (1991) are gathered from different sources. The experimental results may have some discrepancies in terms of some parameters such as size (number of tubes) of the array, length of the array (length of tubes), exact value of the critical flow velocity and so on. In addition to these parameters, the inflow turbulence, the location of the tube in an array, number of tubes used for the investigation, degrees-of-freedom in vibration are known to have an influence on the value of critical flow velocity. The configuration of AMOVI and DIVA experiments contain tube bundles of \(5 \times 5\) and similar to the assumptions made in the theoretical development presented in this article, i.e.
Table 1
Model predictions against the experimental data.

<table>
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<tr>
<th>Array type</th>
<th>Test Seriesa</th>
<th>$p^\circ$</th>
<th>$D$</th>
<th>$m$</th>
<th>$f_p$</th>
<th>$\zeta$</th>
<th>$u_{pc} / u_{pc}^*$</th>
<th>$u_{pc}$</th>
<th>$u_{pc}^*$</th>
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<td>90 (water)</td>
<td>Nak86a (Nakamura et al., 1986)</td>
<td>1.42</td>
<td>19.05</td>
<td>1.1000</td>
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<td>90 (water)</td>
<td>Nak86a (Nakamura et al., 1986)</td>
<td>1.42</td>
<td>19.05</td>
<td>1.1000</td>
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<td>90 (water)</td>
<td>Nak86a (Nakamura et al., 1986)</td>
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<td>19.05</td>
<td>1.1000</td>
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<td>0.2286</td>
<td>1.92</td>
<td>4.15</td>
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<td>90 (air)</td>
<td>Axisa84 (Axisa et al., 1984)</td>
<td>1.44</td>
<td>19.05</td>
<td>0.4920</td>
<td>75.00</td>
<td>1.497</td>
<td>14.197</td>
<td>21.0</td>
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</tr>
<tr>
<td>90 (water)</td>
<td>Pett87 (Pettigrew and Taylor, 1991)</td>
<td>1.22</td>
<td>13.00</td>
<td>0.5400</td>
<td>25.90</td>
<td>0.930</td>
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<tr>
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<td>1.47</td>
<td>13.00</td>
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<td>0.1593</td>
<td>0.87</td>
<td>2.53</td>
</tr>
<tr>
<td>90 (water)</td>
<td>W&amp;AR85 (Abd-Rabbo, 1985)</td>
<td>1.50</td>
<td>25.40</td>
<td>1.2570</td>
<td>16.90</td>
<td>0.590</td>
<td>0.0722</td>
<td>1.05</td>
<td>2.45</td>
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<tr>
<td>90 (water)</td>
<td>Scott7 (Scott, 1967)</td>
<td>1.33</td>
<td>25.40</td>
<td>1.2900</td>
<td>16.20</td>
<td>2.700</td>
<td>0.3393</td>
<td>0.86</td>
<td>2.09</td>
</tr>
<tr>
<td>90 (water)</td>
<td>AMOVI (Cardolaccia and Baj, 2015)</td>
<td>1.44</td>
<td>12.15</td>
<td>0.4523</td>
<td>11.68</td>
<td>1.250</td>
<td>0.2407</td>
<td>0.47</td>
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</tr>
<tr>
<td>90 (water)</td>
<td>DIVA (Shinde et al., 2014)</td>
<td>1.50</td>
<td>30.00</td>
<td>1.4700</td>
<td>18.50</td>
<td>0.920</td>
<td>0.0944</td>
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<td>0.5640</td>
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<td>0.1750</td>
<td>1.10</td>
<td>3.23</td>
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<td>19.05</td>
<td>0.1600</td>
<td>40.00</td>
<td>1.000</td>
<td>0.2010</td>
<td>3.08</td>
<td>4.04</td>
</tr>
<tr>
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<td>1.130</td>
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<td>1.36</td>
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<td>Hal86 (Halle et al., 1986)</td>
<td>1.25</td>
<td>19.05</td>
<td>1.0900</td>
<td>22.90</td>
<td>3.000</td>
<td>0.5660</td>
<td>1.33</td>
<td>3.05</td>
</tr>
<tr>
<td>30 (water)</td>
<td>Hal86 (Halle et al., 1986)</td>
<td>1.25</td>
<td>19.05</td>
<td>1.0900</td>
<td>37.20</td>
<td>3.000</td>
<td>0.5660</td>
<td>2.55</td>
<td>3.60</td>
</tr>
</tbody>
</table>

* Please refer Table 2 of Pettigrew and Taylor (1991) for further details.

Fig. 8. Model predictions of the critical pitch velocity ($u_{pc}$) for (a), (b) a water flow and (c), (d) an air flow experiments.
only the central cylinder is free to oscillate in the lift direction. The instability criteria used in the theory are, first, the amplitude of oscillations becomes half the gap between two adjacent cylinders separated by the pitch and second, the lift component of the Euler number ($E_u$) takes the maximum value based on the pressure drop ($\Delta p_{row}$) across a row of the array. Therefore, the predicted critical velocities may serve as a general threshold of the instability in the array. The critical reduced velocities ($\ast u_{pc}$) predicted by the theory are in a fairly good agreement with the experimental results, especially after considering the different sources of the experimental input parameters.

Fig. 8 shows the variation in the model parameters (Eq. (22)) for increasing value of the arbitrary reduced pitch velocity ($u^*$). Fig. 8(a), (b) show predictions of the critical flow velocity for a water flow experiment (Cardolaccia and Baj, 2015), in which the first plot (Fig. 8(a)) shows a variation of the critical reduced velocity ($u^*_c$) while the second plot shows a variations in the impedance ($I_m$), parameter $\beta$, Euler number ($E_u$) and the fluidelastic proportionality constant ($K_c$) versus the increasing reduced velocity ($u^*$). Similarly, Fig. 8(c), (d) are the model predictions for the air flow experiment (Axisa et al., 1984) listed in the Table 1. In Fig. 8(a), (b) (as well as in (c), (d)) it is evident that the main source of the variation in the critical reduced pitch velocity ($u^*_c$) is the impedance $I_m$ of the system. The Euler number and the proportionality constant vary initially for low Reynolds numbers, however these quantities remain practically constant over a large range of Reynolds number.

A comparison of the theoretical stability boundary for the fluidelastic instability with experimental data for normal-square (90°) tube arrays is shown in Fig. 9(a). The experimental data (shown by using dots) are extracted from a plot in (Paidoussis et al., 2010, Chapter 5), where it is gathered from several sources (cited in the caption of the Fig. 9a). The experimental data shown by +, ×, and □ are for experiments carried out in air for a single cylinder, air for multiple cylinders, water for a single cylinder and water for multiple cylinders respectively. The experimental data represents the stability thresholds for different values of various parameters, such as pitch ratio, Reynolds numbers, stability criteria and so on. The figure also shows the stability boundary predicted by the model for the Reynolds number $Re = 10^5$, pitch ratio $p^* = 1.5$ and critical model parameter $h_c = 50\%$. The trend of the experimental data is very well captured by the model predicted stability boundary (Fig. 9a), particularly the prediction of critical flow velocity at low mass-damping parameter and the discontinuity in the stability boundary for $\approx \frac{m_\delta \rho D}{1^2}$. Figs. 9 show the effect of Reynolds number, pitch ratio and the $h_c$ parameter on the stability boundary respectively. For low Reynolds numbers ($Re < 10^5$), the Euler number increases rapidly (see Fig. 6), which results in decreased values of the proportionality constant $K_c$ (Eq. (23)), consequently the stability boundary predicted by the model decreases for decreasing Reynolds number. Similarly, following Eq. (23), the stability boundaries in Fig. 9c and Fig. 9(d) decrease for the decrease in the pitch ratio ($p^*$) and $h_c$ values. The profile/shape of the stability boundary remains
nearly unchanged for variations in the Reynolds number, $h_c$ and pitch ratio. For the pitch ratio $p_r = 1.5$ and $h_c = 1\%$, the actual displacement of the cylinder is 0.5\% of the cylinder diameter and the stability boundary predicted by the model is well below the experimental data (Fig. 9a) and the lowest curve of Fig. 9(d)).

An important difference between the water flow and air flow test cases in Fig. 8 is the variations in the critical reduced velocity at the onset of the instability. In Fig. 8(a), the arbitrary reduced velocity becomes higher than the estimated critical reduced velocity after $u_{pc}^* = u_{pc}^* = 2.82$. Although with a further increase of the $u_{pc}^*$, the critical reduced velocity $u_{pc}^*$ becomes smaller than the $u_{pc}^*$, indicating the possibility of re-stabilization. On the other hand, the critical reduced velocity ($u_{pc}^*$) in Fig. 8(c) remains almost constant for $\mu > 10$, removing thus possibility of the re-stabilization. The major difference between the two corresponding experiments is the mass ratio ($m^*$), due the difference in the fluid medium. The influence of mass ratio ($m^*$) on the fluidelastic instability thresholds is studied in Tanaka and Takahara (1981) for a normal-square tube array. In their approach, the critical reduced velocity is estimated using the measured unsteady fluidelastic forces on a cylinder of the array. Fig. 10(a) shows the continuous stability boundaries for three values of the logarithmic decrement ($\delta$) and varying mass ratio ($m^*$) for a small ($2 \times 3$) square array with the pitch ratio $p_r = 1.33$. In addition, there are some experimental data points on the stability map (Fig. 10(a)), which are obtained for a bigger ($4 \times 7$) normal-square array with the same pitch ratio ($p_r = 1.33$). The labels ‘A’ and ‘B’ indicate the different modes of vibration. The model (Eq. (22)) prediction of the threshold boundaries for the pitch ratio $p_r = 1.33$ and for different values of the $\delta$ and $m^*$, similar to that of Tanaka and Takahara (1981) as in Fig. 10(a), are shown in Fig. 10(b). Fig. 10(a) and (b) appear to make a good qualitative as well as quantitative match, particularly for the higher values of the mass ratio. The nature of the stability boundaries changes for $u_{pc10}$ in both the stability maps of Fig. 10.
The stability thresholds predicted in Fig. 10(b) by using Eq. (22) are the lower stability limits for a particular value of the logarithmic decrement (δ). As discussed above, the variation in the critical reduced pitch velocity at low mass ratio (low velocities) provides a possibility of the re-stabilization (at least theoretically) for velocities higher than the critical values. In practice, due to nonlinear effects, the re-stabilization is unlikely to occur (Paidoussis et al., 2010, Chapter 5). The actual stability boundaries predicted by the model include the multiple stability thresholds. For example, Fig. 11 shows the stability map for a square array with pitch ratio $p^* = 1.33$. A constant value of the logarithmic decrement is used ($\delta = 0.01$). The red curves represent the unstable boundaries for an increasing value of the velocity, while the green curves represent the re-stabilization boundaries. Further investigation is required for more accurate shapes of the multiple stability boundaries, since they depend on the mechanical impedance term ($I_m$) of the model.

### 4. Model predictions for other array geometry patterns

An extension of the presented theoretical model for the other array geometry patterns is provided in this section. The other three array patterns are, namely, rotated-square ($45^\circ$), normal-triangular ($30^\circ$) and rotated-triangular ($60^\circ$). The theoretical development in Section 2.1 is for a normal-square ($90^\circ$) of equal pitch ratio in both the transverse and longitudinal directions. The longitudinal pitch ratio (now noted as $P_{90}$ for the normal-square array) is considered to be interacting with the perturbations produced by the cylinder as shown in Fig. 2. In the similar manner, the flow through other array types can be idealized as shown in Fig. 12. Figs. 12 show the schematics of interactions between the flow perturbations and array patterns for rotated-square ($45^\circ$), normal-triangular ($30^\circ$) and rotated-triangular ($60^\circ$) respectively. The interstitial flow forms flow channels adjacent to the central cylinder, similar to Fig. 2. Due to the different arrangements of the cylinders, the longitudinal pitch of the flow channels change for the different array types. Let $P_{45}$, $P_{30}$ and $P_{60}$ be the longitudinal pitch of the flow channels for the different array types. Let $P_{45}$, $P_{30}$ and $P_{60}$ be the longitudinal pitch of the flow channels for rotated-square ($45^\circ$), normal-triangular ($30^\circ$) and rotated-triangular ($60^\circ$) respectively. These pitch distances can be expressed as,

<table>
<thead>
<tr>
<th>$p^*$</th>
<th>$Re$ range</th>
<th>$c_0$</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$c_4$</th>
</tr>
</thead>
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<tr>
<td>1.25</td>
<td>$3 \times 10^5$</td>
<td>0.795</td>
<td>0.247 $\times 10^8$</td>
<td>$0.335 \times 10^8$</td>
<td>$-0.155 \times 10^8$</td>
<td>0.241 $\times 10^8$</td>
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<tr>
<td>1.25</td>
<td>$2 \times 10^6$</td>
<td>0.245</td>
<td>0.339 $\times 10^8$</td>
<td>$-0.984 \times 10^8$</td>
<td>0.132 $\times 10^8$</td>
<td>$-0.599 \times 10^8$</td>
</tr>
<tr>
<td>1.50</td>
<td>$3 \times 10^6$</td>
<td>0.683</td>
<td>0.111 $\times 10^8$</td>
<td>$-0.973 \times 10^8$</td>
<td>$-0.426 \times 10^8$</td>
<td>0.574 $\times 10^8$</td>
</tr>
<tr>
<td>1.50</td>
<td>$2 \times 10^6$</td>
<td>0.203</td>
<td>0.248 $\times 10^8$</td>
<td>$-0.758 \times 10^8$</td>
<td>0.104 $\times 10^8$</td>
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<tr>
<td>2.00</td>
<td>$7 \times 10^5$</td>
<td>0.713</td>
<td>0.448 $\times 10^8$</td>
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<td>$-0.582 \times 10^8$</td>
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</tr>
<tr>
<td>2.00</td>
<td>$10^4$</td>
<td>0.343</td>
<td>0.303 $\times 10^8$</td>
<td>$-0.717 \times 10^8$</td>
<td>0.88 $\times 10^7$</td>
<td>$-0.38 \times 10^8$</td>
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<td>$10^5$</td>
<td>0.162</td>
<td>0.181 $\times 10^8$</td>
<td>$0.792 \times 10^8$</td>
<td>$-0.165 \times 10^8$</td>
<td>0.872 $\times 10^8$</td>
</tr>
<tr>
<td>2.00</td>
<td>$5 \times 10^3$</td>
<td>0.33</td>
<td>0.989 $\times 10^8$</td>
<td>$-0.148 \times 10^8$</td>
<td>0.192 $\times 10^8$</td>
<td>$-0.862 \times 10^8$</td>
</tr>
<tr>
<td>2.50</td>
<td>$5 \times 10^3$ to $2 \times 10^5$</td>
<td>0.119</td>
<td>0.498 $\times 10^8$</td>
<td>$-0.507 \times 10^8$</td>
<td>0.251 $\times 10^8$</td>
<td>$-0.463 \times 10^8$</td>
</tr>
</tbody>
</table>

Fig. 13. The pressure drop coefficient vs Reynolds number plot for normal-triangular tube banks. Source: Singh and Soler (1984). ($a, b$ are the pitch ratios in the transverse and longitudinal direction respectively).
The construct of the theoretical model remains the same, except for Eq. (17) and (18). Eqs. (17) and (18) can be restated for rotated-square ($45^\circ$), normal-triangular ($30^\circ$) and rotated-triangular ($60^\circ$) respectively as,

$$P_{A3} = 2\pi \frac{u_p}{\omega_{sh}}, \quad P_{A0} = 2\pi \frac{u_p}{\omega_{sh}}, \quad P_{I0} = 2\pi \frac{u_p}{\omega_{sh}}$$

and

$$\alpha = \frac{\lambda}{P_{A3}}, \quad \alpha = \frac{\lambda}{P_{A0}}, \quad \alpha = \frac{\lambda}{P_{I0}}$$

The model predictions of critical flow velocity for normal-triangular array ($30^\circ$) are tabulated in Table 1 against the experimental data of gathered by Pettigrew and Taylor (1991). The critical reduced velocities are estimated by using $h_e = 50\%$ and the Euler number, which is estimated by means of Eq. (24). The coefficients of Eq. (24) are provided in the top table from Fig. 13. Fig. 13 shows the variation in the Euler number for increasing Reynolds number and various pitch ratio for the staggered equilateral array configuration ($30^\circ$). In general, the critical reduced velocities ($u^*_{cr}$) provided by the experiments and the values predicted by the model show a good agreement.

A comparison of the theoretical stability boundary against a large number experimental data is provided in Fig. 14 for the three types of array. The experimental data shown by +, x, * and □ are for experiments carried out in air for a single cylinder, air for multiple cylinders, water for a single cylinder and water for multiple cylinders respectively, for all the three array types. The theoretical stability boundaries are estimated by for the pitch ratio $p^* = 1.5$, $h_e = 50\%$ and $Re = 10^5$ for normal-triangular array, while $Eu = 0.4$ is used for the rotated-square ($45^\circ$) and the rotated-triangular ($60^\circ$) array configurations due to unavailability of the Euler number vs. Reynolds number data. The source of the experimental data is (Paidoussis et al., 2010, Chapter 5), where the data are gathered from different sources (refer caption of the Fig. 14). The difficulties in such a comparison of large and scattered experimental data with a theoretical model are rightfully addressed in Pettigrew and Taylor (1991) and recently in (Paidoussis et al., 2010, Chapter 5). However, some prominent features of the fluidelastic instability boundaries can be discussed. They include the discontinuity in
the stability boundary between the low and high values of the mass-damping parameter as well as the value of the exponent of the mass-damping parameter ($m\delta \rho D^2$). In general, the model predictions very well follow the experimental data (Figs. 14, 9a) for all types of array and for all range of the mass-damping parameter, especially considering the simplicity of the model. The discontinuity in the stability boundary for $m\delta \rho D^2 > 8 \times 10^{-3}$ is well captures by the presented model. The theoretical models (Tanaka and Takahara, 1980; Tanaka and Takahara, 1981; Chen, 1983; Lever and Weaver, 1986; Price et al., 1990) that include the unsteady fluid forces are able to produced this discontinuity in the stability boundary, which is also evident in the experimental data. In addition to the unsteady interactions of the cylinder and fluid forces, the present model accounts for effects of Reynolds number, pitch ratio and array configuration on the critical reduced velocity. The exponent of the mass-damping parameter becomes $\approx 0.5$ for $m\delta \rho D^2 > 10$, while as for the lower values of mass-damping parameter, the effective value of the exponent decreases; however in this range ($m\delta \rho D^2 < 10$) the mass ratio and damping ratio take different values of exponent (refer Eqs. (22) and (20)).

5. Conclusion

A theoretical model for the fluidelastic instability in cross-flow tube array is presented in this article. The model accounts for dynamic interactions between an oscillating cylinder with the adjacent interstitial flow. Furthermore, the effect of array pattern, pitch ratio and Reynolds number is incorporated in the formulation. The comparison of the critical reduced velocity and stability boundary predicted by the model against a large experimental data is fairly good. The prominent aspects of the fluidelastic instability, such as the mechanisms of instability and possibilities of the multiple stability regions at low mass-damping parameter, discontinuity/jump in the stability boundary for high mass-damping parameter are well captured by the model. The mechanical impedance, which is an equivalence for the fluidelastic damping, can be improved in terms of the exponential decay function based on experimental values of actual perturbation decay; which should lead to improved predictions of the multiple stability boundaries and phase difference between the fluid force and cylinder displacement.

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References


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